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POWER EXPENDED FOR PUMPING HEAT-TRANSFER AGENTS AND EFFICIENCY OF THE HEAT EXCHANGE IN CONICAL RADIATIVE SLOT RECUPERATORS

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Formulas for calculating the amount of power expended for the pumping of heat-transfer agents (air and smoke gases) in conical radiative slot recuperators with account for the initial acceleration of these agents, the acceleration of them in the process of their movement, and their friction on the walls of a channel have been derived. The dependence of the energy coefficient on the flow rate of a heated air, pumped in such a recuperator in the forward-flow and counterflow regimes, and on the recuperator length has been determined on the basis of heat-engineering calculations performed with the use of mathematical models involving the formulas derived.

The energy coefficient of heat exchangers is one of the main criteria of their optimization [1] by the heat-exchange efficiency. It is determined as

$$E_{\text{c}} = Q_{\text{a}}/N_{\Sigma}\,. \tag{1}$$

In engineering calculations, the power expended for the pumping of heat-transfer agents is usually determined by the arithmetic mean values of physical constants of air and smoke gases and by the average velocities of their flows [2]. It is suggested that, in this case, the power is mainly expended for the friction of air on the walls of channels in the units of an apparatus, forming heat-exchange surfaces. However, this approach cannot be used for conical radiative slot recuperators (CRSR) because their design is such that the velocity of the air flowing through the slot between two coaxial conical surfaces in them changes substantially. Therefore, in this case, the amount of power expended for overcoming the inertia of an air can be larger than the amount of power expended for overcoming its friction on the walls of a channel. Moreover, for recuperators of any design, it is necessary to take into account the power expended for the initial acceleration of a heat-transfer agent because the initial velocity of an air flow can be fairly large. Consequently,

$$N_{\Sigma} = N_{\text{in.ac}} + N_{\text{f}} + N_{\text{ac}}\,. \tag{2}$$

The value of $N_{\text{in.ac}}$ is calculated most simply:

$$N_{\text{in.ac}s} = 0.5 m v_0^2\,. \tag{3}$$

To determine $N_{\text{f}} + N_{\text{ac}}$, it is necessary to integrate the equation

$$d\,(N_{\text{f}} + N_{\text{ac}}) = dpV\,, \tag{4}$$

in which dp can be determined from the equation [3]

$$\frac{dp}{dx} = \rho\,\frac{\zeta v^2}{2D} + \rho v\,\frac{dv}{dx}\,. \tag{5}$$

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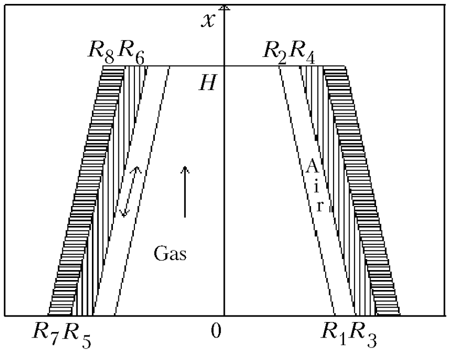

Fig. 1. Diagram of a conical recuperator with a one-way heating and a two-layer thermal insulation: R_1 and R_2 are the radii of the lower and upper bases of the gas channel, R_3 and R_4 are the same radii for the outer wall of the air channel; R_5 and R_6 are the same radii for the outer wall of the second isolation layer.

In the case where the mass flow of a heat-transfer agent $m = \rho v = \text{const}$, Eqs. (4) and (5) are rearranged into the expression

$$\frac{dN}{dx} = m\left(\frac{\zeta v^2}{2D} + v\frac{dv}{dx}\right), \tag{6}$$

where $N = N_{\text{f}} + N_{\text{ac}}$. The quantity ζ is calculated by the Colebrook–White equation [3]

$$\zeta = \left[1.74 - 0.88\ \ln\left(\frac{\varepsilon}{R} + \frac{18.7}{\text{Re}\sqrt{\zeta}}\right)\right]^{-2} \tag{7}$$

($R = D/2$). Our experience in calculations shows that Eq. (7) is well solved by the iteration method; for steel tubes with ζ ranging from 0.01 to 0.05, the accuracy reaches 10^{-5} after 3–4 steps and the relative error of the solution does not exceed 0.1%. In this case, the calculated values of ζ exactly correspond to the diagram presented in Fig. 16.21 in [3]. For the air channel of a recuperator, the value of R is assumed to be equal to the current width of the annular gap between two coaxial conical surfaces, and for the gas channel, this value is assumed to be equal to the current radius of the inner conical surface.

The amount of power expended for the pumping of heat-transfer agents in conical recuperators was determined in the process of their heat-engineering calculations with the use of corresponding mathematical models and programs [4, 5]. The calculations were carried out in the following order. At first, by integrating a system of differential equations, we calculated the temperatures of the air and gas, their velocities, and the Reynolds number as a function of the coordinate x; the values of these parameters were tabulated with a constant step in the memory of a computer. Then Eq. (6) was integrated by the Simpson method because the right side of this equation is the known tabulated function of the coordinate x:

$$N = m\int_0^H \Phi(x)\,dx\,, \tag{8}$$

where

$$\Phi(x) = \frac{\zeta v^2}{4R} + v\frac{dv}{dx}\,.$$

Thus, the total amount of power expended for the pumping of a heat-transfer agent in a CRSR is calculated by the formula

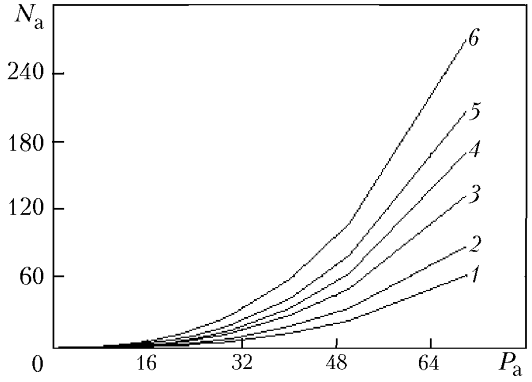

Fig. 2. Dependence of the power expended for the pumping of an air on its flow rate: H = 2 (1, 4), 5 (2, 5), and 10 m (3, 6) [1–3) forward-flow regime, 4–6) counterflow regime]. P_a, 10^3 m^3/h; N_a, kW.

TABLE 1. The Values of the Coefficients in (10)

Coefficients	Forward-flow regime			Counterflow regime		
	H, m					
	2	5	10	2	5	10
A_0	0.03362	0.02597	–0.01512	0.01301	0.01811	0.02812
A_1	–0.17176	–0.18005	–0.06192	–0.06137	–0.08018	–0.08337
A_2	0.38165	0.74968	1.34977	0.12967	0.26663	0.49623
A_3	0.44961	0.50166	0.59771	0.16689	0.22287	0.31964

$$N_\Sigma = m\left[0.5v_0^2 + \int_0^H \Phi(x)\,dx\right]. \tag{9}$$

In the process of using a recuperator, the flow rate of a gas (combustion products) in it cannot be controlled; therefore, the efficiency of the heat exchange in the recuperator can be increased to a maximum value only by changing the rate of an air flow.

Figure 2 presents the dependence of the power expended for the pumping of an air in conical recuperators of height 2, 5, and 10 m operating in the forward-flow and counterflow regimes on the rate of an air flow in them. The recuperators considered were identical in the other geometric parameters: the radii of the lower and upper bases of the inner frustrum of the cone (gas channel) were equal to R_1 = 0.5 m and R_2 = 0.25 m and the corresponding radii of the second (from the center) cone were equal to R_3 = 0.6 m and R_4 = 0.35 m. A heated air was pumped through an annular gap between the first and second cones. The recuperators had two isolation layers located between the second and third cones and between the third and fourth cones. The rate of a gas flow was 5000 m^3/h in all cases. The initial temperature of the air was +20°C and the initial temperature of the gas was +1200°C.

Analysis of Fig. 2 shows that the amount of power expended for the pumping of an air in both the forward-flow and counterflow regimes increases approximately by the cubic law with increase in the air-flow rate; however, the power expended in recuperators operating in the counterflow regime is approximately three times lower than that in recuperators of the same height operating in the forward-flow regime. The graphs presented in Fig. 2 were constructed on the basis of data obtained by the method of least squares. These data show that the dependence $N_a(P_a)$ can be exactly represented by the polynomial

$$N_a(P_a) = A_0 + A_1P_a + A_2P_a^2 + A_3P_a^3. \tag{10}$$

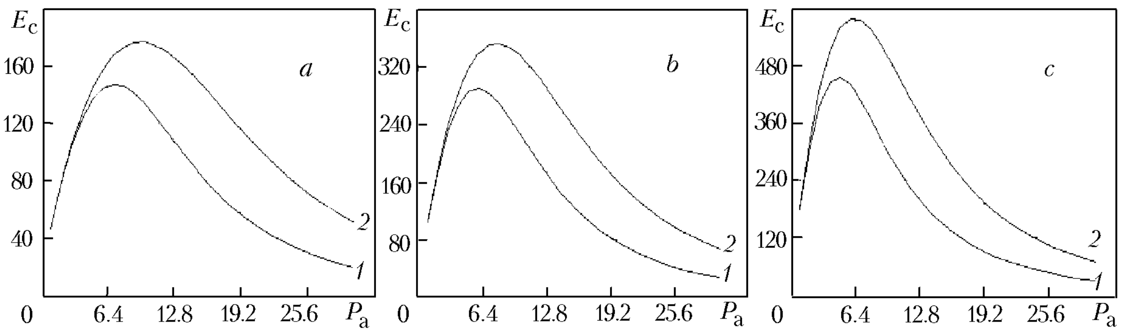

Fig. 3. Dependence of the energy coefficient on the flow rate of an air in recuperators of different height: H = 2 (a), 5 (b), and 10 m (c) [1) counterflow regime, 2) forward-flow regime). P_a, 10^3 m^3/h.

The values of the coefficients in (10) are presented in Table 1.

The residual root-mean-square deviation was not higher than 0.015 kW in all of the cases at a maximum power ranging from 5.5 to 28 kW. Evidently, such behavior of the dependence of the power expended for the pumping of an air on its flow rate will be true for conical and cylindrical radiative slot recuperators with any geometric parameters; only the coefficients in Eq. (10) will be different for different recuperators.

The power expended for the pumping of a gas was equal to ~1 kW in all of the cases considered and was practically independent of the air-flow rate.

Figure 3a presents the dependence of the energy coefficient E_c, determined by formula (1), on the flow rate of a heated air in recuperators of height 2 m. The maximum value of E_c (or the maximum efficiency of the heat exchange in such a recuperator operating in the counterflow regime), equal to 176.8, is attained at an air-flow rate of 10,000 m^3/h. The maximum efficiency of the heat exchange in a recuperator operating in the forward-flow regime, equal to 147.4, is attained at an air-flow rate of 7000 m^3/h. The forward-flow and counterflow regimes are practically identical in efficiency at small air-flow rates, and the efficiency of the counterflow regime is approximately two times higher at large flow rates (20,000 m^3/h and larger).

Figure 3b and c presents analogous dependences for recuperators of height 5 and 10 m respectively. It is seen that, by and large, these dependences are similar to those for the recuperator of height 2 m; however, the efficiency of heat exchange increases and its maximum shifts toward smaller air-flow rates with increase in the recuperator height.

CONCLUSIONS

1. The power expended for the pumping of a smoke gas in a CRSR is small as compared to the power expended for the pumping of a heated air; the latter increases by the cubic law depending on the flow rate of the heated air in recuperators independently of their height and the regime of their operation.

2. Recuperators operating in the forward-flow and counterflow regimes are practically identical in the efficiency of heat exchange, determined by the energy coefficient, at small rates of an air flow in them. At large flow rates, the efficiency of the counterflow regime is approximately two times higher than that of the forward-flow regime.

3. The dependence of the efficiency of CRSRs on the air-flow rate in them has a clearly defined maximum independent of their length and the regime of operation. In this case, the maximum efficiency of the counterflow regime is 25% higher and is attained at an air-flow rate 30% larger than that in the forward-flow regime.

NOTATION

D, effective diameter, m; E_c, energy coefficient; H, height of a recuperator, m; m, mass flow of a heat-transfer agent, kg/sec; N, power expended for the pumping of a heat-transfer agent, W; p, total hydraulic resistance, W/m^3; Q, heat flow, W; R, radius, m; Re, Reynolds number; v, velocity, m/sec; V, current volume of a heat-transfer agent, m^3; x, coordinate on the axis of a recuperator, m; P, volumetric rate of a heat-transfer agent flow, m^3/sec; ε, roughness of the walls of a channel, m; ζ, coefficient of hydrodynamical resistance; ρ, density of a heat-transfer agent, kg/m^3. Subscripts: a, air; in.ac, initial acceleration of a heat-transfer agent; f, friction of a heat-transfer agent on the walls of a channel; ac, acceleration of a heat-transfer agent; 0, value at $x = 0$; Σ, total value.

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